“A new geometry for Einstein’s theory of relativity and beyond”

2nd Workshop

July 7-11, 2025

Faculty of Mathematics, University of Vienna, HS 6, 1st floor
Oskar-Morgenstern-Platz 1, 1090 Wien

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
08:45 - 09:00	*Welcome*				
09:00 - 10:30	*Shin-ichi Ohta*	*Sumati Surya*	*Robert McCann*	*Annegret Burtscher*	*Elefterios Soultanis*
10:30 - 11:00	*Coffee break*				
11:00 - 12:30	*Andrea Mondino*	*Saúl Burgos* *José Luis Flores* *Jónatan Herrera*	*Eric Woolgar*	*Ettore Minguzzi*	*Christina Sormani*
12:30 - 14:00	*Lunch*				
14:00 - 15:30	*Argam Ohanyan*	*Discussion*	*Discussion*	*Discussion*	*Eleni-Alexandra Kontou*
15:30 - 16:00	*Coffee break*				
16:00 - 17:30	*Stacey Harris*	*Michał Eckstein*	*Discussion*	*Carla Cederbaum*	
19:00 -			*Social dinner*		

General information

Lunch

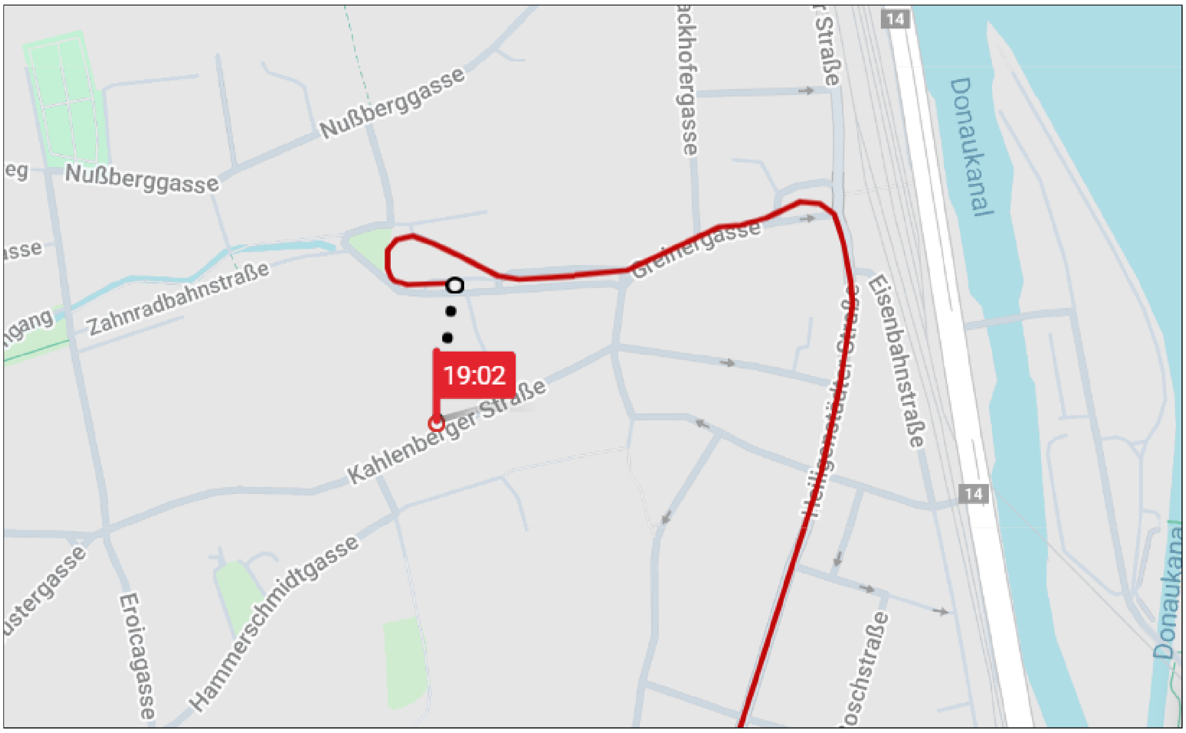
Nearby restaurants:

- *Lia Cafeteria*, Cafeteria of the Faculty of Mathematics, Oskar-Morgenstern-Platz 1
- *Oasia*, Asian Fusion cuisine, Schlickgasse 2
- *Steirasia*, Styrian-Asian Fusion cuisine, Servitengasse 3
- *Gasthaus Rebhuhn*, Typical Austrian cuisine, Berggasse 24
- *Porzellan*, Austrian and international cuisine, Servitengasse 2
- *Cocore*, Italian restaurant, Berggasse 14
- *Pizzeria Riva*, Pizzeria, Türkenstraße 27
- *Allegro*, Italian restaurant, Porzellangasse 21
- *Rembetiko*, Greek restaurant, Porzellangasse 38
- *Fladerei Berggasse*, Stuffed Pita Bread, Berggasse 12

Social dinner, Wednesday 19:00

The social dinner will take place at *Heuriger Schübel-Auer*, Kahlenbergerstraße 22. We will meet at our building at 18:30 and go from there.

If you go by yourself, starting at our building, you can take the tram line D from *Schlick-gasse* in the direction of *Nußdorf, Beethovengang* and get off at the final stop.



Monday, July 7

09:00-10:30

Geometry of Finsler spacetimes

Shin-ichi Ohta

In this talk, I will discuss a Lorentzian analogue to a Finsler manifold, called a Finsler spacetime. For weighted Finsler spacetimes of weighted Ricci curvature bounded below in timelike directions, we can generalize several singularity and comparison theorems, timelike curvature-dimension condition, and timelike splitting theorem (based on collaborations with Yufeng Lu, Ettore Minguzzi, Mathias Braun, Erasmo Caponio, Argam Ohanyan). In the splitting theorem, we are particularly interested in a special class of Berwald spacetimes.

Then, inspired by comparison with the positive definite case, I will explain several open problems: Lorentzian metrization of Berwald spacetimes, characterizations of the concavity of time separation, how to define scalar curvature, etc.

11:00-12:30

On the geometry of synthetic null hypersurfaces and the Null Energy Condition

Andrea Mondino

In the talk, I will present some recent joint work with Fabio Cavalletti (Milan) and Davide Manini (Technion), where we develop a synthetic framework for the geometric and analytic study of null (lightlike) hypersurfaces in non-smooth spacetimes. Drawing from optimal transport and recent advances in Lorentzian geometry and causality theory, we define a synthetic null hypersurface as a triple (H, G, m): H is a closed achronal set in a topological causal space, G is a gauge function encoding affine parametrizations along null generators, and m is a Radon measure serving as a synthetic analog of the rigged measure. This generalizes classical differential geometric structures to potentially singular spacetimes. A central object is the synthetic null energy condition ($\mathsf{NC}^e(N)$), defined via the concavity of an entropy power functional along optimal transport, with parametrization given by the gauge G. This condition is invariant under changes of gauge and measure within natural equivalence classes. It agrees with the classical Null Energy Condition in the smooth setting and it applies to low-regularity spacetimes. A key property of $\mathsf{NC}^e(N)$ is the stability under convergence of synthetic null hypersurfaces, inspired by measured Gromov–Hausdorff convergence. As a first application, we obtain a synthetic version of Hawking's area theorem. Moreover, we extend the celebrated Penrose singularity theorem to continuous spacetimes and we prove the existence of trapped regions in the general setting of topological causal spaces satisfying the synthetic null energy condition.

14:00-15:30

Almost-rigidity of spacetimes with lower timelike sectional curvature bounds

Argam Ohanyan

The celebrated splitting result by Beem-Ehrlich-Markvorsen-Galloway'84 states that a globally hyperbolic spacetime with nonnegative timelike sectional curvature containing a complete timelike line must split that line off isometrically. This result has been generalized in various directions, both to accommodate a weaker curvature assumption in the form of the Strong Energy Condition (due to Eschenburg'88, Galloway'89, Newman'90 and Galloway-Horta'95), as well as to the synthetic setting of Lorentzian length spaces (due to Beran-O.-Rott-Solis'23). In this talk, we present a quantitative version of the above-mentioned splitting theorem in the spirit of the almost-splitting result by Cheeger-Colding'96 in the Riemannian signature. More specifically, we show that if a globally hyperbolic spacetime with almost nonnegative timelike sectional curvature contains a long maximizing timelike segment, then causal diamonds along that segment are close to causal diamonds in product metric spacetimes, where the closeness is framed in terms of the Lorentz-Gromov-Hausdorff distance of bounded Lorentzian metric spaces developed by Minguzzi-Suhr'24 and the error goes to zero as the length of the segment becomes unbounded and the curvature bound below goes to zero. The main ingredients of the proof are timelike semiconcave functions and their gradient flows, and the main purpose of the curvature bound is to ensure that the approximate Busemann functions to and from the endpoints of the long segment are of the former type. At the end, we will touch on open problems in the context of rigidity and almost-rigidity of spacetimes.

This talk is based on ongoing joint work together with Melanie Graf (U. Hamburg), Nicola Gigli (SISSA), Robert McCann (U. Toronto), Eric Woolgar (U. Alberta) and Matteo Zanardini (SISSA).

16:00-17:30

Some approaches to extending and generalizing spacetimes

Stacey Harris

There have been various approaches to address the question, "What happens when spacetime structure breaks down?" These include boundary constructions, low-regularity (or alternative-to-spacetime) approaches to "going beyond the boundary", and low-regularity alternatives to spacetimes generally. I will review some of these and ask what sort of information various approaches are trying to get at.

Tuesday, July 8

09:00-10:30

Causal Set Kinematics: What can discreteness tell us about Lorentzian geometry?

Sumati Surya

In this discussion, I will introduce the causal set approach to spacetime, largely from a kinematic point of view. I will focus on the program of geometric reconstruction, in which continuum geometric and topological invariants are constructed from order invariants of an underlying random causal set, at a given discreteness scale. A key conjecture of the approach, its Hauptvermutung, is that a random causal set corresponds uniquely to an equivalence class of Lorentzian geometries, the latter being determined by the choice of discreteness scale. In this talk I will discuss and explore a proposal for a Taxi-Cab distance function between equivalence classes of Lorentzian geometries, constructed from a class of order invariants of the underlying random causal set. This can in turn be interpreted as a "coarse grained" distance function between Lorentzian spacetimes. An open question is how this ties in with other measures of closeness, in particular versions of the Lorentzian Gromov-Hausdorff distance function.

11:00-12:30

Synthetic approach to the causal boundary: Definition and applications

Saúl Burgos, José Luis Flores, Jónatan Herrera

Since its inception, causal completion has been presented as a less restrictive and more intrinsic construction than conformal completion (more commonly used in relativity). Primarily based on the conformal structure of spacetime, its development has typically been considered from a low regularity perspective (under suitable regularity conditions), and in some cases has been introduced in the context of chronological sets [Harris].

With the increasing interest on a synthetic approach to Lorentzian geometry, it becomes natural to view the causal completion as one of the canonical ways to complete what have come to be known as Lorentzian metric spaces. Furthermore, unlike in the classical approach, in this new paradigm both the original space and its completion live within the same category, allowing for more natural and meaningful comparisons.

In this talk, we will revisit the notion of the causal boundary within the framework of Lorentzian metric spaces, placing particular emphasis on the key differences from the smooth case. We will examine how this new approach allows us to view the c-completion as a Lorentzian metric space in its own right, and explore some of the relationships that arise between the original space and its completion. Finally, we will outline some applications, like a low regularity approach to Bartnik's conjecture, and propose some future research directions.

16:00-17:30

Operational separation of compact regions of spacetime

Michał Eckstein

The causal structure of spacetime induces constraints on any information transfer, forcing it to be subluminal. These constraints are self-evident if we assume that information is encoded in local (classical) physical systems. However, in nonlocal theories — such as quantum mechanics — one needs to carefully apply suitable operational rules of information processing. In the context of Bell-type experiments these "no-signalling constraints" are phrased in terms of conditional probabilities associated with points in the Minkowski spacetime. In my talk I will present a generally covariant formulation of 'no-signalling principle' valid in any causal spacetime. To this end, I will introduce the concept of operational separation between regions of spacetime, which is related but not equivalent to the standard spacelike separation. The established framework allows to grasp some counter-intuitive effects, such as jamming of nonlocal correlations, which hint at a possibility of beyond-quantum nonlocal theories.

Wednesday, July 9

09:00-10:30

Trading linearity for ellipticity: a nonsmooth approach to Einstein's theory of gravity and the Lorentzian splitting theorems

Robert McCann

While Einstein's theory of gravity is formulated in a smooth setting, the celebrated singularity theorems of Hawking and Penrose describe many physical situations in which this smoothness must eventually break down.

In positive-definite signature, there is a highly successful theory of metric and metric-measure geometry which includes Riemannian manifolds as a special case, but permits the extraction of nonsmooth limits under dimension and curvature bounds analogous to the energy conditions in relativity: here sectional curvature is reformulated through triangle comparison, while and Ricci curvature is reformulated using entropic convexity along geodesics of probability measures. This lecture explores recent progress in the development of an analogous theory in Lorentzian signature, whose ultimate goal is to provide a nonsmooth theory of gravity. In particular, we establish a weighted splitting theorem for $g_{ij} \in C^1$ by sacrificing linearity of the d'Alembertian to recover ellipticity. We exploit a negative homogeneity p-d'Alembert operator for this purpose.

The same technique yields a simplified proof of Eschenberg (1988) Galloway (1989) and Newman's (1990) confirmation of Yau's (1982) conjecture, bringing all three Lorentzian splitting results into a framework closer to the Cheeger-Gromoll splitting theorem from Riemannian geometry.

11:00-12:30

Can we prove synthetic positive mass theorems?

Eric Woolgar

Synthetic Ricci curvature bounds and, in Lorentzian language, the Strong Energy Condition have enabled proofs of various Ricci comparison type results such as Myers's theorem, the Cheeger-Gromoll splitting theorem, and the Hawking-Penrose singularity theorem. Riemannian positive mass theorems, however, belong to scalar curvature comparison theory, while positive mass theorems in spacetime use the Dominant Energy Condition.

However, there is a so-called "poor man's positive energy theorem" that use the strong energy condition. There is also one that uses the null energy condition. These theorems might be low hanging fruit for generalization to a synthetic formulation. There's even a positive mass theorem (for asymptotically anti-de Sitter spacetimes) that uses no energy condition at all. I'll review these theorems and pose some interesting side problems as well.

In the spirit of the workshop, this will be a short and speculative talk, with open problems which I hope will lead to an active and productive discussion.

Thursday, July 10

09:00-10:30

Zero, one, and then many negative directions

Annegret Burtscher

In the smooth setting there is no difference in how we define curvature for Riemannian and semi-Riemannian manifolds. When turning to synthetic curvature bounds, however, one has to work already much harder in Lorentzian geometry. The biggest obstacle here is not the curvature per se, but the lack of a canonical metric space structure for the whole space. Still, the causal structure and time separation function, can fill in much of the gap between Riemannian and sufficiently nice Lorentzian manifolds. But how to handle the general semi-Riemannian setting with more than one negative direction? Ideally a natural metric and synthetic extension (or maybe several) should also exist for a subclass of semi-Riemannian manifolds. I will point out some obstacles, naive approaches and ideas, and hope to hear of some more in return.

11:00-12:30

A synthetic version of Lorentzian geometry based on a two-point function

Ettore Minguzzi

In this talk, we introduce the concept of "Lorentzian metric space" — a geometric framework built on a two-point Lorentzian distance function designed to model spacetime under low regularity conditions. We demonstrate how this framework allows for the recovery of chronological and causal relations, as well as a well-behaved Polish topology. Additionally, we discuss key causality properties, including the validity of a limit curve theorem and the existence of time functions. Finally, we explore the close relationship between this notion and global hyperbolicity.

16:00-17:30

Asymptotic notions and questions in GR

Carla Cederbaum

In General Relativity, we consider asymptotically flat (or Minkowskian) spacetimes as well as asymptotically flat (or Euclidean) initial data sets inside such spacetimes. For physical reasons, it is desirable to conceive notions of mass and total (spin) angular momentum in both cases. For initial data sets, it is desirable to split these notions into notions of energy and linear momentum and of center of mass and angular momentum, respectively. When investigating, say, the quantitive rigidity/stability of certain geometric inequalities like the positive mass theorem and the Penrose inequality, it becomes relevant to understand how to extend those notions to non-smooth contexts. I will give an introduction to all the above-mentioned concepts, their physical motivation, and some problems and questions surrounding them.

Friday, July 11

09:00-10:30

Generalized Lorentzian product

Elefterios Soultanis

The construction of a generalized Lorentzian product starts with a (one-parameter) family of metrics on a given space X and produces a Lorentzian length space structure on the product of X and an interval. This construction encompasses all globally hyperbolic spacetimes as well as Lorentzian warped products of Alexander et al. It has a strong flavour of metric geometry and offers a “toy model” to study e.g. time-like Ricci curvature bounds and the non-smooth splitting theorem. After reviewing the construction I will discuss some related open questions including

(1) Causal character of maximizing curves;

(2) Steep time functions and their maximal weak subslopes (a la the octet paper);

(3) Lorentzian Gromov Hausdorff convergence in the context of generalized Lorentzian products.

11:00-12:30

Causally-Null-Compactifiable Space-Times and Convergence

Christina Sormani

In joint work with A Sakovich we introduced the notion of causally-null-compactifiable space-times which can be canonically converted into compact timed-metric-spaces using the cosmological time of Andersson-Howard-Galloway and the null distance of Sormani-Vega. We produced a large class of such space-times including future developments of compact initial data sets and regions which exhaust asymptotically flat space-times. We introduced a variety of notions of convergence of space-times using these notions including the timed-Hausdorff distance. Here we will review the key ideas in our paper (arXiv:https://arxiv.org/pdf/2410.168002410.16800) and we hope to discuss how related notions can be developed using measured convergence and intrinsic flat convergence.

14:00-15:30

Generalizations of classical relativity theorems

Eleni-Alexandra Kontou

Several classical relativity theorems, including the famous singularity theorems, have in their assumptions pointwise energy conditions. Those conditions bound the energy density (or similar quantities) on every spacetime point and are easily violated by quantum fields. One way to examine the applicability of those theorems in semiclassical gravity is to replace them with an averaged version, where the energy density is bounded on a segment of a causal geodesic. The index form method, used instead of the Raychaudhuri equation, provides a direct way of using those weakened conditions. In this talk I will explain how this method applies to several classical relativity theorems, the progress that has been made and the challenges ahead.