“A new geometry for Einstein’s theory of relativity and beyond”

Kick-off Workshop

February 24 - 28, 2025

HS 11, Faculty of Mathematics, University of Vienna
Oskar-Morgenstern-Platz 1, 1090 Wien

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
08:45 - 09:00	*Welcome*				
09:00 - 10:30	*John Harvey*	*Fay Dowker*	*Olaf Müller*	*Eric Ling*	*Sumati Surya*
10:30 - 11:00			*Coffee break*		
11:00 - 12:30	*Tobias Beran, Felix Rott*	*A. Shadi Tahvildar-Zadeh*	*Christian Ketterer*	*Jiří Podolský, Roland Steinbauer*	*Discussion*
12:30 - 14:00			*Lunch*		
14:00 - 15:30	*Nicola Gigli*	*Lawrence Frolov*	*Mathias Braun*	*Leonardo Garcia-Heveling*	*Wolfgang Wieland*
15:30 - 16:00			*Coffee break*		
16:00 - 17:30	*Felix Finster*	*Discussion*	*Davide Manini*	*Discussion*	*Simone Vincini*
19:00 -			*Social dinner*		

General information

Lunch

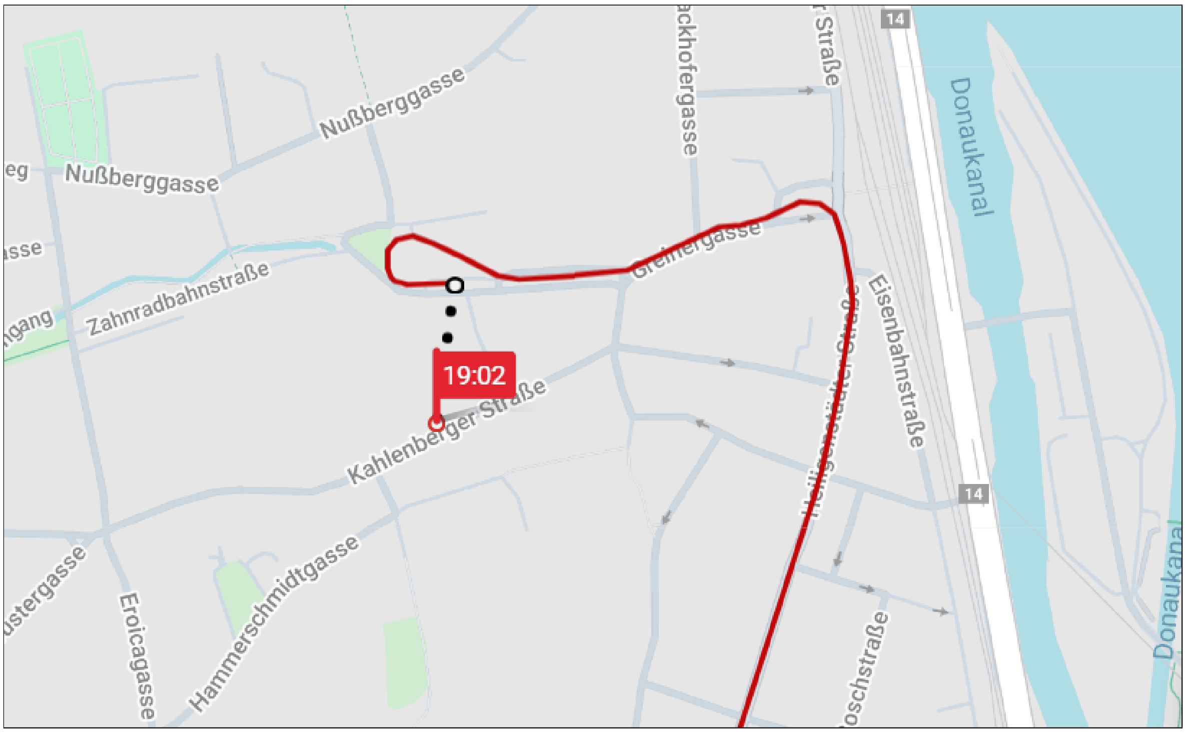
Nearby restaurants:

- *Lia Cafeteria*, Cafeteria of the math faculty, Oskar-Morgenstern-Platz 1
- *Oasia*, Asian Fusion cuisine, Schlickgasse 2
- *Steirasia*, Styrian-Asian Fusion cuisine, Servitengasse 3
- *Gasthaus Rebhuhn*, Typical Austrian cuisine, Berggasse 24
- *Fladerei Berggasse*, Stuffed Pita Bread, Berggasse 12
- *Cocore*, Italian restaurant, Berggasse 14
- *Pizzeria Riva*, Pizzeria, Türkenstraße 27

Social dinner, Wednesday 19:00

The social dinner will take place at *Heuriger Kierlinger*, Kahlenbergerstraße 20. We will meet at our building at 18:30 and go from there.

If you go by yourself, starting at our building, you can take the tram line D from *Schlick-gasse* in the direction of *Nußdorf, Beethovengang* and get off at the final stop.



Monday, February 24

09:00-10:30

Lorentzian length spaces with lower curvature bounds

John Harvey

Length spaces with lower curvature bounds (Alexandrov spaces) have very strong geometric properties. They are stratified by manifolds. They admit a large class of semi-concave functions which have well-defined gradients, and the flow along the gradient field defines a Lipschitz map. The singular strata act as sinks for the gradient flow. Small neighbourhoods are homeomorphic to tangent cones, creating a strong link between the infinitesimal and local structures. All these properties mean that Alexandrov spaces, despite their basic definition, are geometrically very well behaved, not too different from Riemannian manifolds, and admit a rich theory.

This session will be an opportunity to discuss open problems and share progress on Lorentzian length spaces with lower curvature bounds. As generalisations of smooth spacetimes, are they analogous to Alexandrov spaces as generalisations of Riemannian manifolds? Topics will include stratification, gluing, tangent cones and the existence of low-regularity metrics.

11:00-12:30

Current and future advances in Lorentzian spaces with upper curvature bounds

Tobias Beran, Felix Rott

The success of metric geometry in positive signature was an important factor in developing modern synthetic Lorentzian geometry and continues to be a vast source of inspiration for new research in the theory both in analytical and geometric nature. In this talk we want to discuss a number of results - both ongoing and of future interest - in the context of Lorentzian length spaces with curvature bounded above in the sense of timelike triangle comparison which we think would be interesting and desirable to achieve.

In particular, we want to mention and discuss (Lorentzian versions of): A majorisation theorem of Reshetnyak, upper curvature bounds for discrete spaces, a lower bound on the length of timelike curves, characterisation of nonpositive curvature and convex variance, and a Kirszbraun type extension theorem.

14:00-15:30

The concept of directed completion

Nicola Gigli

In positive signature, in order to develop an effective calculus on nonsmooth spaces it is crucial to rely on functional analsysis. The bottom line is that in such framework useful existence results of appropriate 'linear objects' can be found. Because of this, it seems natural to try the same in Lorentzian signature. The idea, though, soon encounters a key obstacle, as the presence of the reverse triangle inequality prevents the presence of any natural Banach space: for instance, there is no natural Banach space of tangent vector fields on a Lorentzian manifold. It seems therefore that, if any, a new sort of functional analysis has to come into play where, among other thing, the concept of Cauchy-completion (that makes no sense without triangle inequality) has to be replaced with a different notion. Our proposal is to concentrate on an order-theoretical form of completion. Among other things, a question is then: can we suitably complete any given partial order, in the same manner that we can complete a metric space? I wish to discuss this topic and its natural links with the classical notion of future infinity. Hopefully, this completion procedure will also be useful in finding 'the right' notion of convergence of non-smooth Lorentzian spaces.

16:00-17:30

Causal fermion systems as an approach to non-smooth Lorentzian geometry

Felix Finster

The theory of causal fermion systems is an approach to fundamental physics. From the mathematical perspective, it provides a general framework for desribing and analyzing non-smooth geometries. Its analytic backbone is the causal action principle, a geometric variational principle which generalizes the Einstein-Hilbert action. Afer a few general remarks I want to focus on a few aspects which seem most relevant in the context of non-smooth Lorentzian geometry:

- spin connection and spin curvature of a causal fermion system
- positive mass, quasi-local mass and scalar curvature
- cone structures
- notions of convergence of sequences of spacetimes

Tuesday, February 25

09:00-10:30

Boundary Lorentzian Geometry from Discrete Order

Fay Dowker

There are good reasons to accept that a Lorentzian geometry can be encoded in a causal set aka a discrete order aka a transitive directed acyclic graph. I will give a very brief review of these reasons. Then I will discuss the geometry of spacelike, timelike and null boundaries of a spacetime region and whether boundary geometry can also be encoded in a causal set. I aim to focus on open problems for the purposes of discussion and collaboration.

11:00-12:30

Classical and Quantum Laws of Motion for Singularities of Spacetime

A. Shadi Tahvildar-Zadeh

I will briefly explain the necessary conditions under which worldlines of charged particles can be identified with time-like singularities of spacetime, and show examples of classical as well as quantum laws of motion for them. I will elaborate on the crucial role that the geometry of spacetime in the neighborhood of these singularities plays in formulating these laws, and explain why I believe that the development of a theory of weak (and weaker!) solutions for Einstein's equations is a prerequisite for the ultimate success of this endeavor.

14:00-15:30

The initial value problem for a massless scalar field and N point-charges in one space dimension

Lawrence Frolov

In this talk we discuss recent advancements in the study of point charges jointly evolving with a coupled field or space-time metric in one space dimension. Historically, the joint evolution for point charges and their sourced field is inconsistent due to divergent self-force terms appearing in the particle's equation of motion. Resolving this inconsistency has been an open problem for over a century, until a recent breakthrough was made by Kiessling. He showed that under certain regularity conditions on the fields, an admissible force law can be derived from the assumption of local energy-momentum conservation. We will apply Kiessling's method to prove that well-posedness of the joint IVP for several point charges and their sourced field holds in one space dimension if and only if a compatibility condition holds between the initial field data and initial particle data. Unexpectedly, this compatibility condition implies that nearly all physical properties of the point particles, namely their initial positions, velocities, and charges are uniquely determined by the initial field data. The open discussion will focus on how to extend these results to the one-dimensional general relativistic setting.

Wednesday, February 26

9:00-10:30

Maximality and Cauchy developments in synthetic Lorentzian geometry

Olaf Müller

We explore the notion of Cauchy development in the context of Lorentzian length spaces and show that, assuming a simple and natural analogy to the metric measure case, whose proof is work in progress, we find maximal Cauchy developments that are proper extensions of the maximal Kruskal spacetime.

11:00-12:30

Generalized cones and Ricci curvature bounds

Christian Ketterer

In this talk I will present new results on synthetic lower Ricci curvature bounds for generalized cones over a metric measure space. Here, a generalized cone can be understood as a metric measure space or as measured Lorentzian length space. More precisely, I will present conditions on the warping functions and on the fiber space that imply a measure contraction property. This is a joint work with Clemens Sämann and Matteo Calisti.

I will explain some of the key features of generalized cones and warped products in general, and I will explain ideas that enter the proof of our result in particular. Furthermore, I will present and discuss a couple of open problems in relation to warped product constructions. Most importantly, we conjecture that the assumptions in our theorem not just give a measure contraction property but the full curvature-dimension condition. A simpler form of the problem in the context of metric measure spaces is the so-called tensorization problem for the curvature-dimension condition that is also still open in the general case, assuming that spaces are nonbranching and but not necessarily infinitesimally Hilbertian.

14:00-15:30

New perspectives on the d'Alembertian from general relativity

Mathias Braun

This presentation has multiple objectives. First, we motivate and review a new distributional notion of the d'Alembertian from mathematical relativity, more precisely, a nonlinear p-version thereof, where p is a nonzero number less than one. This operator comes from natural Lagrangian actions introduced relatively recently. Unlike its classical linear yet hyperbolic counterpart, it is nonlinear yet has elliptic characteristics. Second, we describe recent comparison estimates for the p-d'Alembertian of Lorentz distance functions (notably a point or a spacelike hypersurface). Their new contribution implied by prior works on optimal transport through spacetime is a control of the timelike cut locus. Third, we illustrate exact representation formulas for these p-d'Alembertians employing methods from convex geometry. Fourth, several applications and open problems are presented. Partly in collaboration with Tobias Beran, Matteo Calisti, Nicola Gigli, Robert McCann, Argam Ohanyan, Felix Rott and Clemens Sämann.

16:00-17:30

Null Energy Condition in a fully non-smooth setting

Davide Manini

Recently, several attempts have been done in order to develop a Lorentzian analogous of the Curvature-Dimension (CD) condition in Riemannian signature. The aim of these contributions is to give a fully non-smooth formulation of General Relativity. In this sense, a satisfactory Time-like Curvature-Dimension (TCD) condition has been proposed; roughly speaking, TCD means that the Ricci curvature has a lower bound on time-like vectors.

However, the Null Energy Condition (NEC), that is the fact that Ricci curvature is non-negative on light-like vectors, still lacks a fully non-smooth formulation. One of the reasons is that the time displacement in light-like directions is degenerate. Thus it is not possible to give a non-smooth definition of light-like geodesics which enjoy the same properties available in Riemannian signature or in time-like directions.

In this talk, I will propose a method for overcoming this issue, namely, by giving a selection geodesics in a causal space. I will discuss strengths and weaknesses of this approach.

Thursday, February 27

09:00-10:30

C^0-inextendibility of spacetimes: known results and possible future directions

Eric Ling

A spacetime M is C^0-extendible if there is a spacetime M', with a continuous metric, such that M embeds isometrically as a proper open subset within M'. If no such continuous extension exists, then M is said to be C^0-inextendible. Determining whether or not a given spacetime is C^0-inextendible has proven to be a challenging mathematical problem. The study of C^0-inextendibility was initiated by Jan Sbierski in 2015 when he proved that the maximally analytic Schwarzschild spacetime is C^0-inextendible. Since then other results have been found, but our understanding of C^0-inextendibility is far from complete. In this session, we will review known results and present possible future directions in this study.

11:00-12:30

Impulsive gravitational waves: past and future

Jiří Podolský, Roland Steinbauer

Impulsive gravitational waves are spacetimes that model short but strong bursts of gravitational radiation. They have been introduced by Roger Penrose in the mid 1960s and have since been much studied by researchers in exact solutions, who have widely extended the original class of models, by particle physicists, who have studied (quantum) scattering in these backgrounds, and mathematicians who have used these geometries as key-examples of spacetimes of low regularity. Indeed, impulsive gravitational waves can be described by a locally Lipschitz continuous Lorentzian metric but can also be transformed (formally) into a form which is better adapted to the symmetries of the background spacetime. This, however, comes at the price of lowering the regularity of the metric to be merely distributional and the notorious 'discontinuous coordinate transformation' has only recently been understood mathematically in the general non-exanding case.

In this introductory talk we discuss construction methods for impulsive waves as well as the geodesics in these spacetimes. We intend to provide the background for ongoing and future studies that look at impulsive waves from the perspective of

(a) spacetime matching using the new null-matching formalism due to Mars-Manzano, which allows to generalise Penrose's original cut and paste approach to impulses with pressure, and

(b) synthetic Lorentzian geometry which investigate sectional and Ricci curvature bounds of these geometries.

14:00-15:30

How to affinely parametrize null geodesics?

Leonardo Garcia-Heveling

Null geodesics play an important role in Lorentzian geometry, especially in Penrose's singularity theorem, which predicts null geodesic incompleteness. While the affine parametrization of a timelike geodesic has a geometrical and a physical interpretation as length and as proper time, this is not the case for a null geodesic. The affine parametrization of a null geodesic is usually defined in terms of the geodesic equation, which is problematic for low-regularity and synthetic approaches. Recently, McCann has sugegsted to instead interpret null geodesics as limits of timelike ones. In this session, we will discuss in depth how this gives rise to notions of null geodesic incompleteness, and where the caveats lie.

Based on joint work with Melanie Graf.

Friday, February 28

09:00-10:30

Some Open Questions in Causal Set Theory

Sumati Surya

In the causal set approach to quantum gravity, continuum spacetime is thought to be an approximation of a random, discrete partially ordered set, or causal set. The continuum geometry is thus encoded in the discrete order theoretic structure, as evident from several geometric and topological reconstruction results. Random causal sets therefore provide an interesting, extreme arena for understanding the role of the causal structure in low regularity spacetimes. In this talk I will outline some of the open questions in causal set theory, of broad relevance to the overarching theme of this conference.

14:00-15:30

Quantum gravity from the quantization of the light cone

Wolfgang Wieland

In classical field theory, a state of the system is characterized by initial data on a Cauchy surface. At the quantum level, different states correspond to different (normalised) states in the Hilbert space of the theory. In gauge theories, there is a further complication: there are gauge symmetries and points on the same gauge orbit represent the same physical state of the system. In vacuum general relativity, the relevant gauge symmetries are diffeomorphisms. Loosely speaking, depending on which point on a gauge orbit we choose, we land on different Cauchy hypersurfaces. In my talk, I will present a research programme in quantum gravity based on the idea to use null hypersurfaces rather than spacelike hypersurfaces to characterize initial data and thereby the physical states of the theory. I will report on three new results on this frontier. First, I provide a non-perturbative characterization of the gravitational phase space on a light cone. Second, the description is taken to the quantum level. Third, an immediate physical implication is found: in the model, the Planck luminosity separates the eigenvalues of the radiated gravitational wave power. Below the Planck power, the spectrum of the radiated power is discrete. Above the Planck power, the spectrum is continuous and the resulting physical states contain caustics that can spoil the semi-classical limit. The talk is based on arXiv:2402.12578, arXiv:2401.17491, arXiv:2104.05803.

16:00-17:30

Gradient flows in Lorentzian spaces

Simone Vincini

Thanks to their intrinsic stability properties and adaptability to rough settings, gradient flows have proven to be a valuable tool in the study of Riemannian manifolds and metric measure spaces. For instance, the identification of the heat flow with the metric gradient flow of the entropy functional has lead to important stability results such as the stability of the RCD condition.

Even though such a tool would prove useful also in the Lorentzian case, no "rough" definition of gradient flow is available yet, not even on the Minkowski space. In this session, I would like to suggest some approaches in this direction and discuss some of the challenges that need to be addressed. Along the way, I will present some formal computations and rigorous results which suggest that some link between some gradient flow on the Lorentzian-Wasserstein space and the p-heat flows is in place.

The results presented in this talk are part of ongoing projects with Mathias Braun, Nicola Gigli, Robert McCann and Matteo Zanardini.